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## Modeling and Analysis of an Electromagnetic Energy Harvester

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### Abstract

*Vibration Energy Harvesters* (VEHs) are devices capable of converting energy from an external source of vibration into electrical energy, which can power small equipment, such as sensors and vibration monitoring instruments. Nowadays, the vast majority of such transducers can be classified into four main categories according to their operating principle, namely: Piezoelectric; electromagnetic; electrostatic and magnetostrictive harvesters. This paper presents the analysis and modeling of a compact commercial device of approximately  $50.97\text{ cm}^3$  in volume and a mass of 90 g, containing a resonant-type electromagnetic generator in its interior. The harvesting power of this device is up to 45 mW, when operating at its resonance frequency of approximately 60 Hz and at an acceleration amplitude of  $0.7g$  [ $\text{m/s}^2$ ], resulting in a normalized power density around  $1.8018\text{ mW/cm}^3/\text{g}^2$ . The objective of this paper is, therefore, to detail the modeling of such a transducer via *Comsol Multiphysics*, as well as the validation of the developed models with the experimental results.

**Keywords:** Energy Harvesting, modeling, electromagnetic, transducer, vibration.

**PACS:** 43.38.Dv – Electromagnetic and electrodynamic transducers.



## 1. INTRODUCTION

*Vibration Energy Harvesting* (VEH) consists of the study and modeling of devices capable of harnessing and converting any form of vibration into electrical energy, which can be used to power sensors or small-scale equipment.

Within this line of research, it is possible to highlight the development of many types of transducers and, especially in the last two decades, most of these devices fall into four main categories, each with its own characteristics, advantages and disadvantages: Piezoelectric; electromagnetic; electrostatic and magnetostrictive *Energy Harvesters* (EHs) [1–3]. The operating principle of these four types of generators, however, is based exclusively on the conversion of mechanical energy from vibrations of any nature into electrical energy.

Piezoelectric devices, for example, make use of specific materials (such as crystals in general) that accumulate electrical charges and generate a potential difference in their interior, when subjected to deformation due to some external load [4].

Electromagnetic transducers, in turn, can generate electricity through the relative movement between a magnet and a coil, from which an electromotive force emerges and is capable of charging batteries or small capacitors by connecting the coil to an external circuit [5].

In this paper, a commercial *Electromagnetic Energy Harvester* (EMEH) is studied and modeled via *Finite Elements Method* (FEM). This device contains a resonant-type linear alternator capable of generating up to 45 mW at its operating frequency of 60 Hz and at an acceleration amplitude of  $0.7g$  [ $m/s^2$ ]. The numerical analysis of such transducer is performed via software *Comsol Multiphysics*, where both *Structural Mechanics* and *Magnetic Fields* physics modules were applied to model its electromechanical behavior in the time and frequency domains. The numerical results were then validated with experimental data, provided in terms of output power per acceleration in the frequency domain.

## 2. ELECTROMECHANICAL MODEL

The main purpose of this section is to analyze both mechanical and electromagnetic characteristics of the *Vibration Energy Harvester* (VEH) illustrated in Figure 1:

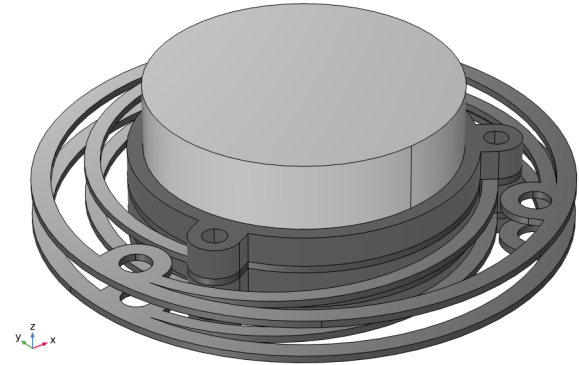


**Figure 1:** 3D geometry of a commercial VEH.

This compact linear generator is basically composed by a seismic mass mounted on a mechanical spring, the latter consisting of six thin, curved beams connected to a pair of internal and external metallic rings. In addition to the mechanical structure, a multi-turn cylindrical coil is positioned around the magnet and attached to the vibrating base together with the outer rings, thus being responsible of converting the external vibration into electrical energy.

### 2.1 2D Equivalent Model

As most of the mechanical components of the electromagnetic oscillator are symmetrical with respect to the vertical axis, it becomes natural to consider a geometric simplification in which the metallic rings are replaced by a pair of thin disks of equivalent stiffness. This way, it is possible to consider a 2D equivalent model representing the mechanical behavior of the original geometry, as detailed in Figure 2.

#### 2.1.1 Components and Materials

Table 1 shows the main components of the transducer, as well as its constituent materials and their main mechanical properties (Young's modulus, density and Poisson ratio, respectively). The seismic mass, in this case, is assembled from three main components: A flat cylindrical magnet made of neodymium-ferrite (NdFeB, grade

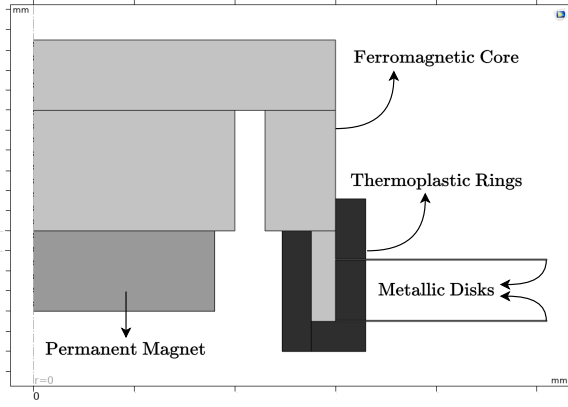


**Figure 2:** 2D axial-symmetric geometry.

N52); a 409 ferritic stainless steel core and *High Density Polyethylene* (HDPE) rings connecting the core to the mechanical springs, which are composed of low carbon steel.

### 2.1.2 Dynamic Analysis of the Transducer

Since most of the transducer mass is concentrated in the seismic mass and the stiffness of the metallic springs is much lower than that of the other components, an equivalent system of one *Degree of Freedom* (DoF) can be considered to describe its dynamic behavior when harmonic excitation is applied to the generator base (in this case, the outer rings). In Figure 3, it is shown the corresponding free-body diagram for such system, where the magnet's vertical displacement over time is represented by  $x(t)$ .

Here, the total mass  $m$  is evaluated by adding the mass of each component; the equivalent stiffness  $k$  is calculated from a modal analysis and later used to perform the thickness adjustment of the metallic disks [6]; and the damping coefficient  $c$  is obtained from a structural damping model, whose loss factor  $\eta$  can be estimated from the experimental results. Furthermore, the base ex-

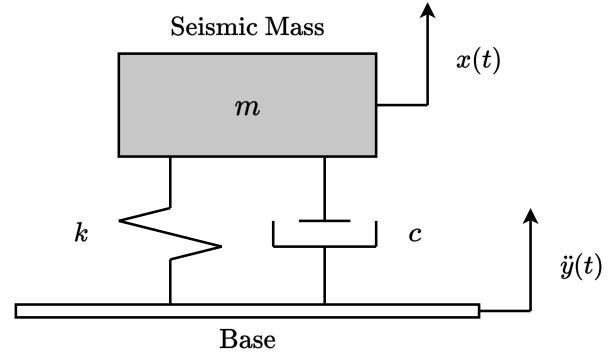


**Figure 3:** Simplified model – 1 DoF system.

citation is given by a harmonic acceleration of the form  $\ddot{y}(t) = Y_{\max} \sin \omega t$ , where the amplitude  $Y_{\max}$  can be written in terms of the gravity's acceleration  $g = 9.80655 \text{ m/s}^2$ . Now, for a harmonic base excitation  $y(t)$ , one can express the relative displacement between the motion of the seismic mass  $x(t)$  and the vibrating base by introducing the variable  $z(t) = x(t) - y(t)$ . Therefore, by setting  $x > y > 0$  and  $\dot{x} > \dot{y} > 0$ , the motion equation of the mechanical system becomes:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0. \quad (1)$$

Rewriting Equation 1 in terms of  $z(t)$ , we have:

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}. \quad (2)$$

For a structural damping model, Equation 2 further simplifies into:

$$m\ddot{z} + \underbrace{k(1 + i\eta)}_{\tilde{k}} z = -m\ddot{y}, \quad (3)$$

where  $\tilde{k} = k(1 + i\eta)$  is often referred as complex stiffness.

Equation 3 shows that the simplified model illustrated in Figure 3 is equivalent to a 1 DoF system

**Table 1:** Constituent materials of the VEH components and its properties.

| Components         | Materials           | Mechanical Properties |                             |       |
|--------------------|---------------------|-----------------------|-----------------------------|-------|
|                    |                     | E [GPa]               | $\rho$ [kg/m <sup>3</sup> ] | $\nu$ |
| Permanent Magnet   | NdFeB N52           | 160                   | 7500                        | 0.24  |
| Ferromagnetic Core | 409 Stainless Steel | 195                   | 7610                        | 0.28  |
| Polymeric Rings    | HDPE                | 1.86                  | 950                         | 0.46  |
| Metallic Rings     | Low Carbon Steel    | 210                   | 7840                        | 0.30  |



where the base is fixed and the seismic mass is subjected to a body force given by  $-m\ddot{y}(t)$ , as detailed in Figure 4.

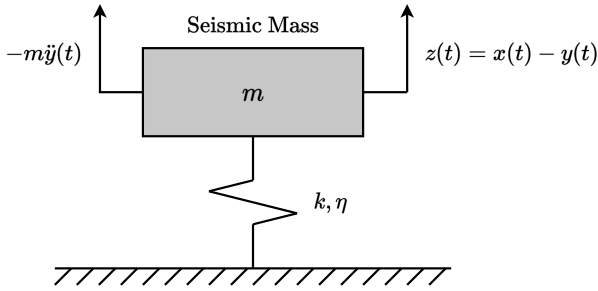


**Figure 4:** Free-body diagram in terms of  $z(t)$ .

Assuming now that the motion of both magnet and base are harmonic, the relative displacement can be expressed as  $z(t) = Z \sin(\omega t - \phi)$ , where  $Z$  and  $\phi$  represent the amplitude and phase of  $z$ , respectively. By plugging  $\ddot{y}(t)$  and  $z(t)$  into Equation 3, one can show that [7]:

$$G = \frac{Z}{Y_{\max}} = \frac{1/\omega_n^2}{\sqrt{(1-r^2)^2 + \eta^2}}. \quad (4)$$

In Equation 4,  $G(\omega)$  is usually referred as a *Frequency Response Function* (FRF) in terms of displacement per acceleration;  $\omega = 2\pi f$  is the vibration frequency of the base in rad/s;  $\omega_n = \sqrt{\frac{k}{m}} = 2\pi f_n$  is the resonance frequency of the transducer and  $r = \frac{\omega}{\omega_n}$  is the ratio between the excitation frequency and  $\omega_n$ . As an immediate result, the maximum displacement amplitude for the seismic mass occurs when  $\omega = \omega_n$  or, in other words,  $G$  is maximum at resonance frequency and its value depends on  $\eta$  and  $\omega_n$  only. In fact,  $G = 1/(\eta\omega_n^2)$  at  $r = 1$ .

### 2.1.3 Electromagnetic Study

To proceed with the electromechanical analysis, it is necessary to define the coil geometry/parameters and the magnetic properties of the materials constituting the seismic mass. Figure 5 illustrates both the mechanical components and coil, as well as the air domain surrounding the entire geometry.

The multi-turn coil consists of  $N = 1600$  turns of AWG41 copper wire (cross-section area of approximately  $0.00396 \text{ mm}^2$ ) and is placed in between the magnet and the ferromagnetic core.

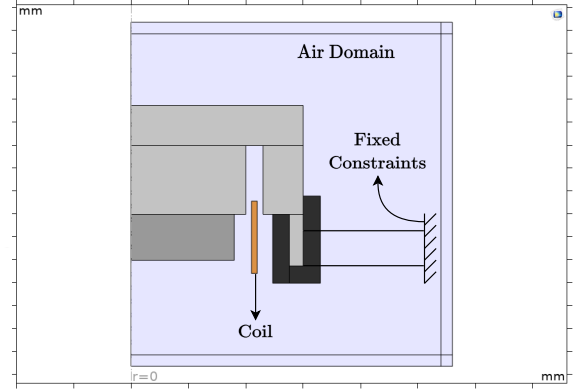


**Figure 5:** 2D axisymmetric geometry with constraints and air domain.

The permanent magnet is made of N52 grade neodymium-ferrite, such that its remanent flux density norm reaches a value of  $\|B_r\| = 1.44 \text{ T}$ , with a recoil permeability of  $\mu_{\text{rec}} = 1.05$ . Thus, the magnetic flux density  $B$  is numerically evaluated inside the magnet's domain as a function of the magnetic field  $H$ :  $B = \mu_0 \mu_{\text{rec}} H + B_r$ , where  $\mu_0 \approx 4\pi \times 10^{-7} \text{ H/m}$  is the magnetic permeability of free space and  $B_r$  is a vector with norm  $\|B_r\|$  and pointing to the positive vertical direction, meaning that the magnet's north pole is the upper half of its geometry.

For the ferromagnetic core, the magnetic induction  $B$  is obtained by considering the B-H curve of 409 stainless steel, where the value of  $B$  inside the armature geometry is numerically evaluated through the relation  $B = f(\|H\|) \frac{H}{\|H\|}$ , where  $f(\|H\|)$  is a function of the applied magnetic field that accounts for the nonlinear behavior of the core's magnetization.

Lastly, the relative permeability of both thermoplastic and metallic rings are considered in the magnetization model. In this case, the flux density is directly proportional to the magnetic field:  $B = \mu_0 \mu_r H$ . For the HDPE rings, the relative permeability is unitary ( $\mu_r = 1$ ). The metallic rings, on the other hand, are composed of low carbon steel, which is a ferritic material. The relative permeability, in this case, is around  $\mu_r = 100$ .

Now, from the Maxwell-Faraday equation, it is possible to estimate the electric field  $E$  from the time-varying magnetic field  $B$  inside the coil, the latter as a result of the relative motion due to

external vibrations.

$$\underbrace{\oint_{\partial\Sigma} E \cdot dl}_{\varepsilon} = - \underbrace{\frac{d}{dt} \left[ \iint_{\Sigma} B \cdot dS \right]}_{\dot{\Phi}_B}. \quad (5)$$

Mathematically, the left side of Equation 5 expresses the spatial and time variation of  $E$  as a line integral around a closed loop  $\partial\Sigma$ , which in this case coincides with the coil circumference. The right side of the equation, in turn, represents the time variation of the magnetic induction  $B$  passing through the coil as a surface integral around the area  $\Sigma$  enclosed by its geometry. Moreover, it is possible to deduce the Faraday-Lenz law of induction from Equation 5, where the variation of magnetic flux  $\Phi_B$  through the surface enclosed by the coil generates an electromotive force  $\varepsilon$  at the coil terminals, given by [8]:

$$\varepsilon(t) - N \frac{d\Phi_B}{dt} = -N \frac{d\Phi_B}{dz} \frac{dz}{dt}. \quad (6)$$

Notice that, in Equation 6, the electromotive force varies with time only and is proportional to the number of coil turns  $N$ ; to the spatial change in magnetic flux  $\frac{d\Phi_B}{dz}$  and to the relative velocity between the magnet and the coil, given by  $\frac{dz}{dt}$ . The current  $I$  flowing through the coil is, therefore, directly proportional to the induced voltage  $\varepsilon$  and inversely proportional to a resistive load  $R_L$  connected to the coil's terminals. In fact, from Ohm's law, we have that  $I = \varepsilon/R_L$ .

It is common in literature, when analyzing the efficiency of Energy Harvesting devices, to define a measure called *Normalized Power Density* (NPD), which correlates the output power of the transducer to its occupied volume and to the vibration's acceleration amplitude in terms of the gravity's acceleration  $g$ :

$$\text{NPD} = \frac{\text{Power}}{\text{Volume} \times (\text{Acceleration})^2}. \quad (7)$$

Observe that the value of the NPD increases when a transducer is capable of generating more power for a given volume and acceleration amplitude. The application of this measurement to the electromagnetic generator detailed in this paper is presented in the next section.

### 3. RESULTS

In this section, the results referring to the electromechanical modeling of the device illustrated in Figure 1 and its 2D equivalent geometry are presented.

#### 3.1 Mechanical Model

Given that the specific mass and the volume surrounding all of the components are known, one can easily calculate the total mass of the mechanical model illustrated in Figure 2. In fact, the total mass and volume of the transducer are, respectively,  $m \approx 90$  g and  $\text{Vol} \approx 50.97$  cm<sup>3</sup>.

Once known the materials, geometry and boundary conditions, the first mode of vibration of the 3D geometry was numerically determined via modal analysis:

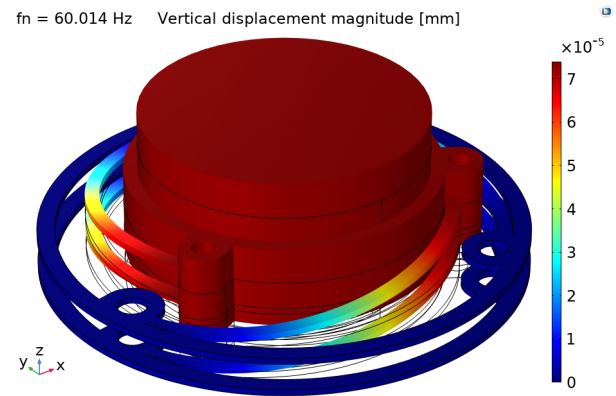


Figure 6: First mode shape – 3D geometry.

Figure 6 shows the first mode shape at the resonance frequency of  $f_n = 60.014$  Hz. As expected, the seismic mass movement occurred exclusively in the vertical direction, thus constituting a transversal mode shape. Also, from this analysis and from the fact that the resonance frequency is, by definition,  $\omega_n = \sqrt{k/m}$ , it is possible to estimate the equivalent stiffness of the metallic springs:  $k \approx 12796.98$  N/m.

Applying the same analysis to the 2D axisymmetric geometry, it was possible to determine the thickness  $h \approx 0.486$  mm of the metallic disks so that the natural frequency maintained at  $f_n = 60.014$  Hz, as shown in Figure 7.

Furthermore, assuming that the system is excited by a sinusoidal acceleration  $\ddot{y}(t)$  with an ampli-

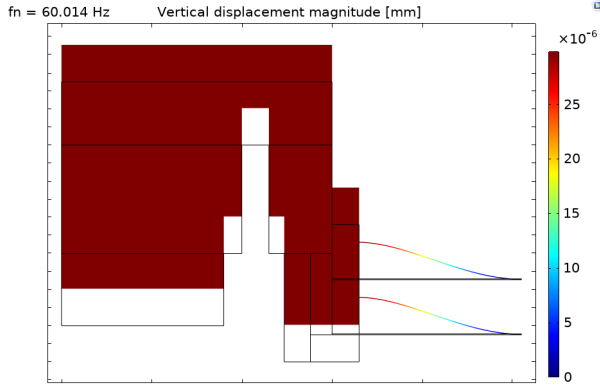


**Figure 7:** First mode shape – 2D axisymmetric geometry.

tude of  $Y_{\max} = 0.7g \text{ [m/s}^2\text{]}$ , one can measure the magnet's vertical displacement as a function of the frequency via mode-superposition method:

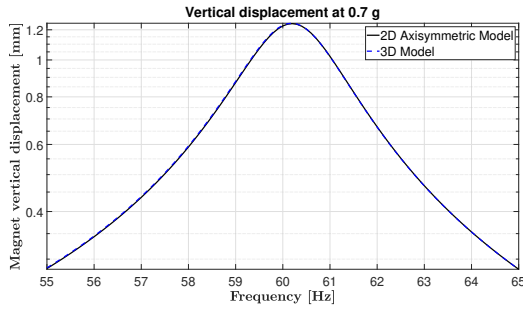


**Figure 8:** Magnet's vertical displacement  $\times$  Frequency.

Figure 8 shows the amplitude of the magnet's displacement in the vertical direction for both 3D and 2D axisymmetric geometries, respectively. The maximum displacement occurred at the resonance frequency, reaching about  $Z \approx 1.2 \text{ mm}$  of amplitude with respect to the fixed edges of the disks. This result is in agreement with the relative displacement amplitude predicted by Equation 4.

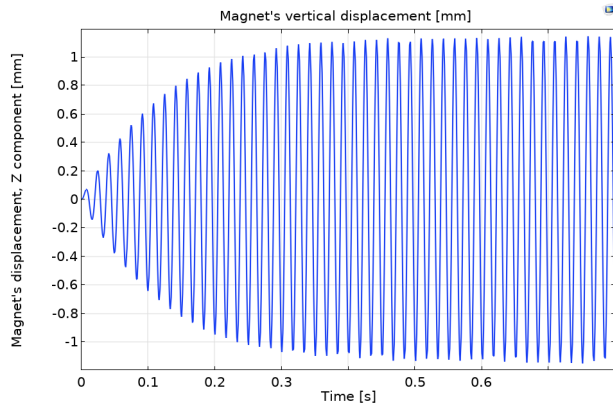


**Figure 9:** Magnet's vertical displacement  $\times$  Time.

In Figure 9, one can notice the asymptotic behavior of the magnet's displacement in vertical direction over time at resonant frequency  $f_n = 60.014 \text{ Hz}$ . The vertical displacement of the magnet stabilizes around 1.17 mm after about 0.5 s and asymptotically approaches the amplitude of 1.2 mm, where it reaches steady state. To perform this transient analysis, a Time Dependent Study was carried out, where a harmonic body force of the form  $m\ddot{y}(t)$  was applied.

### 3.2 Electromagnetic Model

As a result of a Stationary Study performed via FEM, it was possible to numerically determine the flux density norm for both 2D and 3D geometries, as illustrated in Figures 10 and 11, respectively:

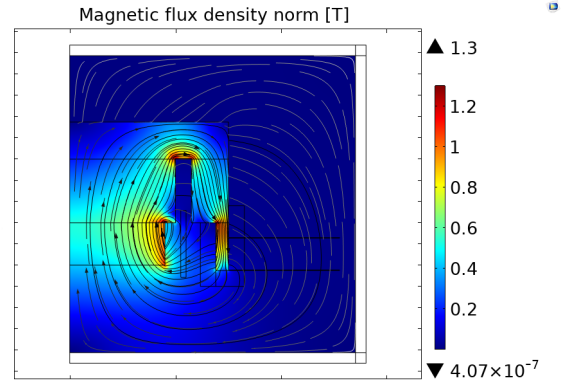


**Figure 10:** Magnetic flux density norm – 2D geometry.

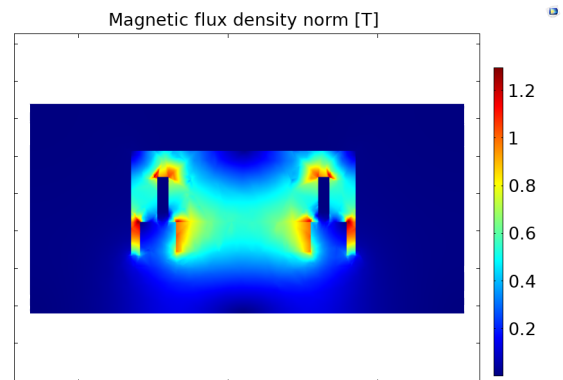


**Figure 11:** Magnetic flux density norm – 3D geometry.

Observe that the ferromagnetic core concentrates the magnetic flux lines around the coil, so that both flux distribution and the term  $\frac{d\Phi_B}{dz}$  in Equation 6 are maximized in the region  $\Sigma$  enclosed by the coil's geometry.

Once the stationary solution for the magnetic induction was obtained, a Frequency Domain Perturbation study was applied to obtain the electromotive force as a function of the excitation frequency. As shown in Figure 12, both the induced voltage at the coil terminals and the current through the coil reach maximum values of  $\varepsilon \approx 6.5 \text{ V}$  and  $I = \varepsilon/R_L \approx 14.77 \text{ mA}$  at the resonance frequency  $f_n$ , respectively. Consequently, the maximum resulting power is about  $P_{\text{coil}} = \frac{1}{2} \text{Re} \{ \varepsilon I^* \} \approx 45 \text{ mW}$ .

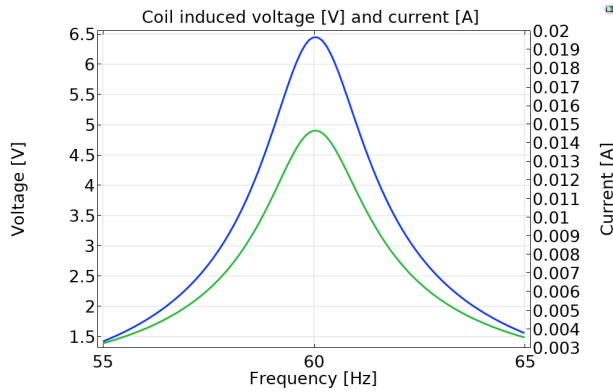


**Figure 12:** Coil induced voltage and current  $\times$  Frequency.

Moreover, for an acceleration amplitude of  $0.7g$ , the maximum expected power takes place at an optimal load resistance of  $R_L \approx 440 \Omega$  (see Figure 13). For larger values of  $R_L$ , the induced current decreases at a faster rate than the increase in induced voltage. A similar observation can be made by decreasing the value of the resistive load connected to the coil terminals.

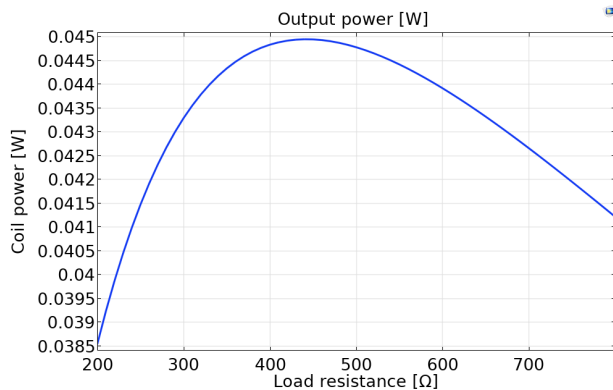


**Figure 13:** Output power  $\times$  Load resistance  $R_L$ .

Now, for an acceleration amplitude of  $0.7g$ , the maximum value of output power results in a maximized NPD of approximately  $1.8018 \text{ mW/cm}^3/\text{g}^2$ .

In other words, this device is capable of generating almost  $2 \text{ mW per cm}^3$  at the second power of the ambient acceleration  $g$ .

### 3.3 Experimental Validation

Figure 14 compares the normalized output power calculated from the 2D equivalent model with the normalized power obtained experimentally. Here, an acceleration amplitude of  $Y_{\text{max}} = 0.1g \text{ [m/s}^2\text{]}$  is applied to the base of the transducer.

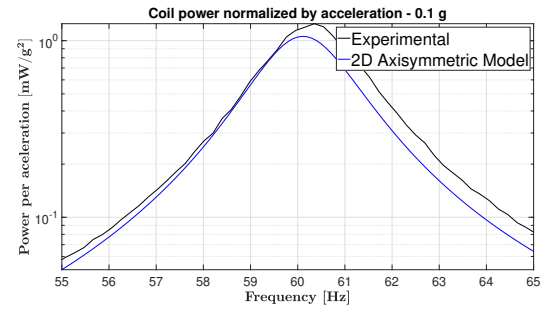


**Figure 14:** Output normalized power – 2D numerical model  $\times$  Experimental data.

From the experimental curve given in Figure 14, one can measure the equivalent loss factor  $\eta$  by using the half-power bandwidth method, since, at frequencies close to the resonance frequency  $f_n$ , the loss factor satisfies the following relation:  $\eta \approx 2\zeta = \frac{f_{\text{upper}} - f_{\text{lower}}}{f_n}$ . Here,  $\zeta$  is the equivalent viscous damping ratio of the system and  $f_{\text{upper}}$ ,  $f_{\text{lower}}$  are the frequencies where the normalized power reaches half of its maximum value. By estimating  $f_{\text{upper}}$ ,  $f_{\text{lower}}$  and applying this method, the loss factor was calculated as, approximately,  $\eta \approx 0.0386$ . This value of  $\eta$  validates the calculation used to predict the relative displacement amplitude  $Z$  in Equation 4.

## 4. FINAL CONSIDERATIONS

The study proposed in this article focused on the modeling of a linear electromagnetic transducer capable of converting and generating electrical energy from mechanical vibrations at its operating frequency of  $60 \text{ Hz}$ . Such generator was capable of delivering an electrical power of approximately  $45 \text{ mW}$  at an acceleration of  $0.7g$ , in which its NPD reached maximum value of  $1.8018 \text{ mW/cm}^3/\text{g}^2$ . One can conclude that this compact device achieved a relatively high value



in terms of NPD, when compared to previously reported harvesters in the literature.

As for future work, the next step consists of optimizing and maximizing the NPD of the energy harvester detailed in this paper. To do so, different design constraints will be taken into account, such as the volume occupied by its final geometry, as well as constructive aspects of each component.

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