

FIA 2020/22

XII CONGRESSO/CONGRESO IBEROAMERICANO DE ACÚSTICA
XXIX ENCONTRO DA SOCIEDADE BRASILEIRA DE ACÚSTICA - SOBRAC

Florianópolis, SC, Brasil

Application of wave-number approach for room behaviour analysis and absorption coefficient measurement

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Abstract

With advances in signal processing technology and processing power of personal computers, new methods have been developed in the area of room acoustics. Using a robotic arm to hold a microphone during measurements, a virtual automated microphone array can be formed, as an alternative to rigid arrays. This type of system can be used for measuring impulse responses related to each microphone position, which in turn can be used as input data to calculate the wave-number spectrum of the sound field. An analysis was performed using the virtual automated microphone array in a laboratory room at UTFPR Curitiba Campus, prepared with a 10.8 m² glass wool overlay. Using the wave-number approach, the reconstruction of the room transfer functions was calculated for a set of 30 validation points, and the average error in the reconstruction processing was calculated and averaged in third-of-octave bands. The error is limited under 10%. The absorption coefficient calculated shows the expected behaviour for the type of material, albeit lightly underestimated. The underestimated values are most likely due to the array characteristics in the frequency range analysed.

Keywords: room acoustics, room impulse response, plane wave decomposition, microphone array, sound field reconstruction.

PACS: 43.55.Ev, 43.55.Mc.



1. INTRODUCTION

The advances and improvements in signal processing technology allow for new techniques [1–5] to be developed in the study of the sound field. One of these new analyses is a technique called plane-wave decomposition, which consists of an inverse problem derived from a spatial Fourier transform with approximation of its base functions by plane waves [3, 5].

In practical terms, the input data for the inverse problem will be the sound pressure or the room transfer function (RTF) measured in several positions. The use of microphone arrays allows for the sampling of the sound field in different points of the room. Aiming for more flexibility, a robotic arm can be used to form a sequenced microphone array. The use of this type of measurement mitigates potential problems usually identified in arrays such as position errors [2] and transducer mismatch [6]. Furthermore, sequenced arrays are capable of reproducing different array formats using the same equipment, such as random measurement positions, which in turn improve the sampling capabilities of the system [2].

A common application of plane-wave decomposition is in calculating the absorption coefficient of a certain surface. The main advantage of this method in this context is circumventing the requirement for diffuse sound field conditions [2], avoiding the systematic errors caused by the usual non-conformities to this requirement, and therefore being suited as the starting point for the development of *in situ* applications [3].

Therefore, this paper describes an experiment involving the calculation of the wave-number (angular) spectrum around the array in the vicinity of a surface covered with an absorbing material. The error between measured and reconstructed RTF is quantified. From the wave-number spectrum, the absorption of the surface per third of octave band is calculated.

2. THEORY

This section is dedicated for a brief review of the techniques and concepts that are useful for

understanding this paper.

2.1 Plane-wave decomposition

Plane-wave decomposition or wave-number approach is a method that represents the sound field with relation to its wave-number [3]. This type of procedure is the basis of recent studies such as the experimental characterization of the sound field in reverberating environments [2], spatial energy decay evaluations [1, 4], amongst others.

The wave-number spectrum (angular) comes from the premise that the sound field around a certain point with position $\mathbf{r}_m = (x_m, y_m, z_m)$ can be represented by the overlapping of plane waves with each direction $\mathbf{k} = (k_x, k_y, k_z)$, which is known as the wave-number vector [3]. Thus, applying an Inverse Three-dimensional Spatial Fourier Transform on the the representation of the sound field by plane waves, the result is Equation (1) [2]

$$p(\mathbf{r}_m) = \iiint_{-\infty}^{+\infty} P(\mathbf{k}) e^{-j(k_x x + k_y y + k_z z)} d\mathbf{k}, \quad (1)$$

in which $p(\mathbf{r}_m)$ is the complex amplitude sound pressure at the point $\mathbf{r}_m$, and $P(\mathbf{k})$ is the wave-number (or angular) spectrum [3]. An interesting remark is that $e^{-j(k_x x + k_y y + k_z z)} = e^{-j\mathbf{k} \cdot \mathbf{r}}$. The wave-number spectrum $P(\mathbf{k})$ can be represented following Equation (2)

$$P(\mathbf{k}) = |P(\mathbf{k})| e^{j\phi(\mathbf{k})}, \quad (2)$$

in which $|P(\mathbf{k})|$ is the absolute value of the spectrum and $\phi(\mathbf{k})$ is its phase. It's worth remarking that the time dependency was omitted, as is common when using complex numbers to represent waves.

In practice, a discrete approximation between $p(\mathbf{r}_m)$ and the wave-number spectrum is employed [2], as described on Equation (3)

$$p(\mathbf{r}_m) \approx \sum_{l=1}^L \tilde{P}(\mathbf{k}_l) e^{-j\mathbf{k}_l \cdot \mathbf{r}_m}, \quad (3)$$

in which $\tilde{P}(\mathbf{k}_l)$ is the wave-number spectrum at wave-number $\mathbf{k}_l$, l is the index that identifies each plane wave that forms the acoustic field,

and L is the number of plane waves (which can be interpreted as the resolution of the plane wave representation of a real sound field). This approximation stems from the fact that all plane waves should be contained on the surface of a radiation sphere [2], indicating the presence of exclusively propagating waves.

The relation exposed in Equation (3) is valid for a single transducer. For an array of microphones, a matrix equation can be formulated [2], such as in Equation (4)

$$\mathbf{p} = \begin{pmatrix} e^{-j\mathbf{k}_1 \cdot \mathbf{r}_1} & \dots & e^{-j\mathbf{k}_L \cdot \mathbf{r}_1} \\ \vdots & \ddots & \vdots \\ e^{-j\mathbf{k}_1 \cdot \mathbf{r}_M} & \dots & e^{-j\mathbf{k}_L \cdot \mathbf{r}_M} \end{pmatrix} \begin{pmatrix} \tilde{P}(\mathbf{k}_1) \\ \vdots \\ \tilde{P}(\mathbf{k}_L) \end{pmatrix}, \quad (4)$$

which is essentially the matrix expansion of Equation (3), $\mathbf{p}$ being a sound pressure vector (expressed as a complex number) with length M corresponding to the amount of discrete positions where $p(\mathbf{r}_m)$ is measured.

It is common to rewrite Equation (4) in a simpler manner, following Equation (5)

$$\mathbf{p} = \mathbf{H}\mathbf{x}, \quad (5)$$

in which $\mathbf{H}$ is the complex $M \times L$ sensing matrix containing $e^{-j\mathbf{k}_l \cdot \mathbf{r}_m}$ [2], and $\mathbf{x}$ is the L length vector containing $\tilde{P}(\mathbf{k}_l)$.

Bringing the reader's attention to Equation (5), and considering that

- $\mathbf{p}$ can be obtained by performing measurements in M positions; and
- $\mathbf{H}$ can be calculated by knowing the M positions and defining the L plane waves;

it becomes clear that to find the $\tilde{P}(\mathbf{k}_l)$ is the same as to solve Equation (5) for $\mathbf{x}$. This solution, however, is not trivial, since in most cases Equation (4) is an ill-posed problem (which means that the matrix $\mathbf{H}$ is rectangular and thus cannot be inverted). Therefore, the solution of Equation (5) for $\mathbf{x}$ requires methods such as regularized least squares method, with the usual choice being Tikhonov regularization for environments where

reverberation is predominant [1, 3–5]. By employing this technique, an appropriate estimation of $\mathbf{x}$ can be found, denoted by $\hat{\mathbf{x}}$.

The inverse problem stated in this research has some limitations, the biggest of them being the exclusion of evanescent waves. These decaying components can be essential when analyzing the near field of sources and scatterers. On the other hand, the evanescent components are more easily corrupted by measurement noise. One advantage of their exclusion is that one obtains an inverse problem that is easier to regularize. In any case, using a regularized solution makes it possible to measure the sound field with an array with arbitrary geometry since there are no mathematical restrictions on the positions of the M points (only hardware limitations) [2]. Another advantage is to explore other regularization methods for the type of expected sound field in the room [3].

2.1.1 Calculating the incidence-dependent absorption coefficient using the wave-number approach

The solution of the inverse problem described on Equation (5) for $\mathbf{x}$ is denoted by $\hat{\mathbf{x}}$, whose components are $\tilde{P}(\mathbf{k})$. Recalling that $\mathbf{k}$ indicates the direction of propagation of each individual plane-wave, a graphic representation of $|P(\mathbf{k})|$ over the sphere with positions given by (k_x, k_y, k_z) can be plotted [2]. It's worthwhile to remark that this spherical graphic can be split into two hemispheres. The lower hemisphere contains the directions where sound impinges in an absorbing sample and the upper hemisphere contains the directions where the sound is reflected by the sample [2]. Thus, $\tilde{P}(\mathbf{k})$ can be written as a function of θ and ϕ , respectively, elevation and azimuth angles (relative to the plane of the surface of the sample).

The absorption coefficient is determined (in terms of either absorbed and incident power or reflected and incident power) through Equation (6)

$$\alpha(\theta) = \frac{W_{abs}(\theta)}{W_{inc}(\theta)} = 1 - \frac{W_{ref}(\theta)}{W_{inc}(\theta)}, \quad (6)$$

where $\alpha(\theta)$ is the absorption coefficient,



$W_{abs}(\theta)$ is the averaged absorbed power, $W_{inc}(\theta)$ is the averaged incident power, and $W_{ref}(\theta)$ is the average reflected power, the latter three as functions only of the incidence angle, by means of averaging in the azimuth. It's important to underline the need for azimuth averaging, as it avoids bias from angles with no incident power [2].

The part of Equation (6) pertaining the absorption term and the ratio between reflected and incident powers can be rewritten now in terms of the wave-number spectrum, by means of Equation (7), according to Nolan *et al.* [2]

$$\alpha(\theta) = 1 - \frac{\int_0^{2\pi} |\tilde{P}(\theta, \phi)|^2 d\phi}{\int_0^{2\pi} |\tilde{P}(\pi - \theta, \phi)|^2 d\phi}, \quad (7)$$

in which the averaging in the azimuth in the interval $[0, 2\pi]$ is evidenced.

2.1.2 Reconstruction of the sound field using plane wave decomposition

Another application of plane wave decomposition is the reconstruction of the sound field at an arbitrary region by means of the estimated solution $\hat{\mathbf{x}}$. What that means is that, from any array from which $\hat{\mathbf{x}}$ can be calculated, a mapping of sound pressure, sound velocity, sound intensity and room transfer function can be found for regions not appertaining to the original array [2, 3, 5]. The reconstruction of the sound pressure and particle velocity in the surface of the absorbing sample can be used as another means to calculate the absorption coefficient of the sample.

Reconstruction of the sound pressure is represented by Equation (8) [5]

$$\mathbf{p}_r = \mathbf{H}_r \hat{\mathbf{x}}, \quad (8)$$

in which $\mathbf{p}_r$ is the size K vector containing the reconstructed sound pressures in K arbitrary positions in the vicinity of the array, and $\mathbf{H}_r$ is the $K \times L$ reconstruction matrix, calculated from the plane wave functions evaluated on the points where the sound field will be reconstructed.

3. EXPERIMENTAL SETUP

All impulse response measurements were done in the EK-019 (Robotics Laboratory) room at UTFPR - Ecoville Campus. The room is a common laboratory room, with no previous acoustic treatment done. The dimensions of the room are 11,8 m $\times$ 5,3 m $\times$ 3,3 m and all procedures were done with furniture on the room resembling a possible *in situ* condition.

The microphone arrays were formed using a Mitsubishi RV-M1 robotic arm that positioned the microphone in predetermined positions. The microphone was attached to the robot's tool flange by a 3D printed holder composed by a base, an extended tubular structure and a coupling part where the microphone is slotted at a 45° angle with respect to the last joint of the robot ("wrist roll"). The holder, depicted in Figure 1, was designed to cause the least amount of impact on the impulse response measurements and to make use of all the degrees-of-freedom the Mitsubishi RV-M1 has available.



Figure 1: Picture of the designed microphone holder.

The Mitsubishi RV-M1 is a five joint robot originally for didactic applications, which meant a certain amount of care was needed when defining the positions, as certain regions could be beyond the machine's reach or dexterity. The relative wrist position of the robot was calculated for each array position and verified using inverse kinematics.

The measurement of the room impulse responses was done with the ITA Toolbox [7] for MATLAB, which already has the measurement procedures implemented. The measurement method selected

was the linear deconvolution. A GROM omnidirectional source was used to excite the room with a frequency sweep, and a GRAS 40AE free field measurement microphone was used to record the sweep in the room. It should be stated that the response level [dB] *versus* frequency [Hz] curve of this microphone is flat until about 10000 Hz (with the correction for direct incidence) [8]. Furthermore, the array is beyond the calculated critical distance [9] of the room (approximately 1.2 m, measured with relation to the source).

A glass wool absorbing sample of 10.8 m² was placed in one of the walls of the room to serve as the surface of interest. The robot was placed so that the point closest to the absorbing surface would be at a distance of 300 mm from the wool. Figure 2 shows the covered surface of interest and the Mitsubishi RV-M1 robot.



Figure 2: Glass wool absorbing surface and Mitsubishi RV-M1 robotic arm.

According to Brandão [10], glass wool is a very porous mineral wool, with its flow resistivity increasing with its density. Thus, the expected behaviour relating to its sound absorption, based on the preexisting models is of an accentuation of absorption following the increase in the frequency of the sound exciting the room.

The measurements for the plane wave decomposition procedure were done using an array with 150 randomly generated positions inside a 400 mm × 100 mm × 170 mm rectangular volume. From the 150 measurement

points, 120 were used to calculate the wave-number spectrum, while the remaining 30 microphone positions were used for the validation of the inverse problem solution. Figure 3 shows the positions of the array (green dots and magenta dots for the plane-wave decomposition and validation respectively) and the robot reference (0,0,0) (yellow dot). All measurements were done with the tip of the microphone facing the interest surface. Each position had its resulting inverse kinematics verified using MATLAB.

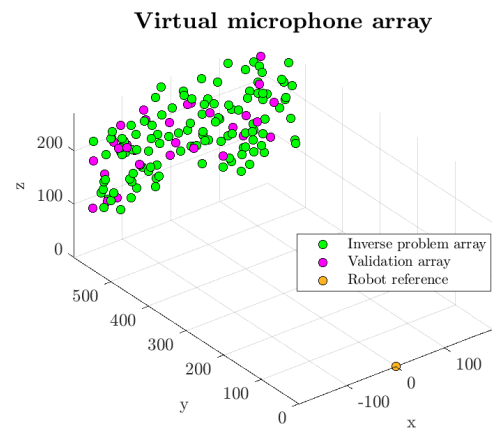


Figure 3: Virtual random array of microphones. Green dots: points measured for the inverse problem. Magenta dots: points measured for validation. Yellow dot: robot reference.

4. RESULTS AND DISCUSSIONS

The wave-number spectrum was calculated using a basis of 2562 plane wave functions, generated over a sphere by means of geodesic subdivision of icosahedron, with the RTF measured at 120 points. The Tikhonov regularization was performed for each desired frequency, with its regularization parameter being chosen using the L-curve criterion [11].

The RTF for the 30 validation points were reconstructed, and the relative percentage error between the reconstructed RTF and the measured RTF at each point was calculated following Equation (9)

$$\varepsilon_{\%} = 100 \times \left| \frac{V_R(f) - V_M(f)}{V_M(f)} \right|, \quad (9)$$

in which $V_R(f)$ is the reconstructed RTF in a



certain point (as a function of the frequency f), $V_M(f)$ is the measured RTF in the same point.

Figure 4 shows the reconstruction (blue solid line) and the measured RTF (green dashed line) between 224 Hz and 2820 Hz at the validation position number 30, as an example.

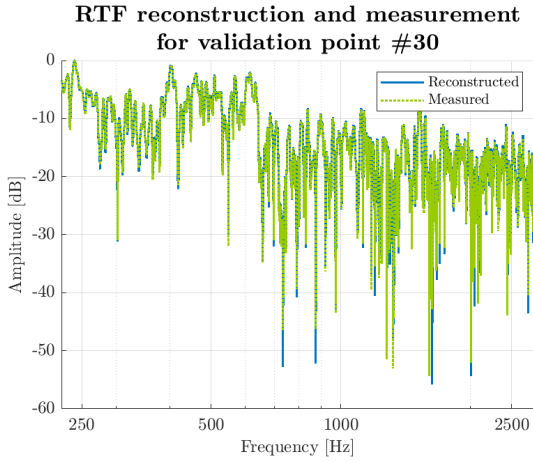


Figure 4: Comparison between the reconstructed RTF and the measured RTF at validation point number 30 between 224 Hz and 2820 Hz. Blue solid line: reconstruction. Green dashed line: measurement.

The coherence between the reconstruction and the measurement indicates that both the algorithm for the plane-wave decomposition and the RTF reconstruction are valid. Moreover, the relative percent error between the reconstructed RTF and the measured RTF with relation to the measured RTF was also calculated, by averaging the errors in the validation points and then averaging in the third of octave bands. Figure 5 displays the average reconstruction error for the third-of-octave bands (maroon asterisks, with the dashed line added to avoid parallax).

Notice that between the 400 Hz and the 1000 Hz bands the error is limited under 2.5 %, indicating good coherence for the reconstruction inside this frequency range. For the 250 Hz and 315 Hz frequency bands there is a slight rise on the error, most likely due to the dimensions of the array and their influence on spatial resolution of the array [12]. Nevertheless, the error is still relatively small, bounded under 5 %. For the 1250 Hz and 1600 Hz bands there is also an increase in the error, still limited under 5 %, and for the higher analysed frequency bands – 2000 Hz and 2500 Hz, the error extrapolates to just short of

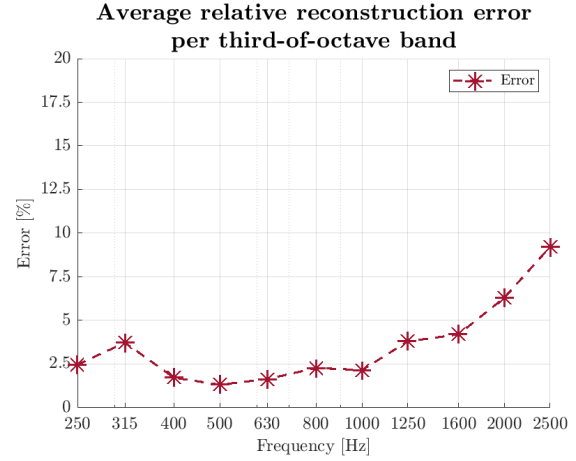


Figure 5: Average relative reconstruction error from 250 Hz band to 2500 Hz band.

10%. The increase in the error in the higher frequencies are related to the spacing between microphones [12], which was not controlled as of this experiment.

After the array was determined valid, the wave-number spectrum estimations can be analysed. Figure 6 shows a tri-dimensional energy distribution in the region of the array at approximately 969 Hz. The color scale indicates the energy level (in dB) of $\tilde{P}(\theta, \phi)$ with relation to its largest value, from yellow (higher values) to blue (smaller values). In this representation, the k_z axis is in the direction perpendicular to the absorbing sample, the k_y axis points to the ceiling of the room, and the k_x axis points to the right side of the sample. The dynamic range was limited in -15 dB, to facilitate the identification of strong reflections.

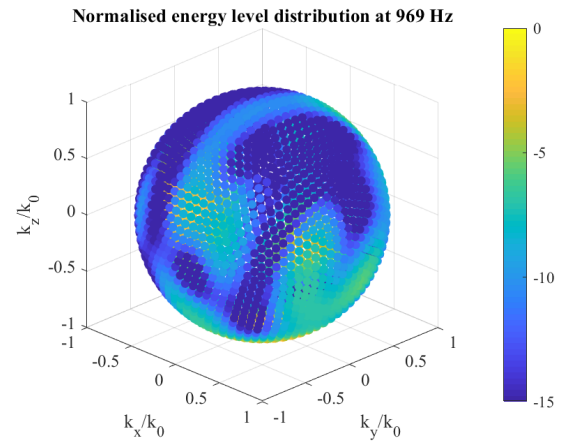


Figure 6: Normalised energy level distribution around the array's vicinity at 969 Hz.

Due to the tri-dimensional shape of the graphic it can be hard to visualize all the incidences and reflections simultaneously outside the MATLAB environment, so it is usual to represent this energy distribution as an azimuth-elevation map. Moreover, due to the discretization of the spectrum over the sphere being homogeneously distributed on its surface, it is interesting to interpolate the calculated wave-number to an evenly spaced grid, which is convenient for the numerical integrations and averages that will be performed [12]. Figure 7 shows an interpolated plane mapping of the same wave-number spectrum from Figure 6. The interpolation was performed with an angular step of $\pi/60 \text{ rad} = 3^\circ$.

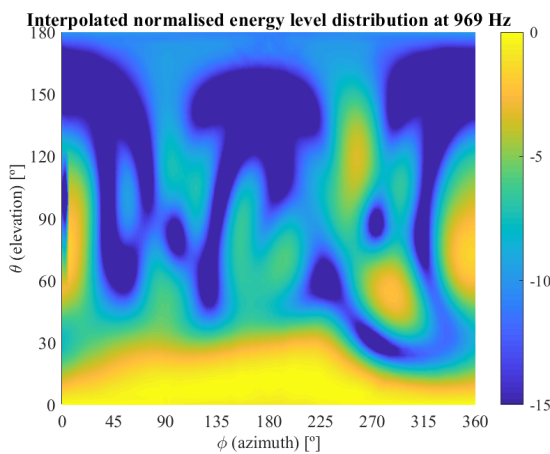


Figure 7: Interpolated plane mapping of the energy distribution at 969 Hz.

In Figure 7, some reflections and incidences can be identified. The light blue region through the whole extension of the azimuth ϕ in the elevation θ between 150° and 180° is the reflection of the interest surface. The yellow region on the elevation 0° is the direct radiation from the source. Other reflections can be observed between an elevation of 45° and 150° , and are reflections caused by other surfaces and irregularities of the room such as furniture, the ceiling, the floor and the robot's pedestal.

Another analysis that can be performed is the evaluation of the absorption of the surface with the increase in sound frequency. The calculation of the absorption can be done by calculating the absorption coefficient using Equation (7) using the average of the interpolated wave-number

spectrum in third-of-octave bands. Nolan [12] cites that it is important to use the average in third-of-octave bands to avoid biases due to non-isotropic energy distribution for pure tones. Posterior averaging in the azimuth angle ϕ leads to a value of absorption dependent on the incidence angle θ . This can be used to calculate the diffuse incidence absorption (α_s) by means of Equation (10) (known as Paris Formula) [10]

$$\alpha_s = \int_0^{\pi/2} \alpha(\theta) \sin(2\theta) d\theta. \quad (10)$$

Figure 8 shows the diffuse incidence absorption calculated by means of the Equation (10) (blue asterisks with dashed line added to avoid parallax).

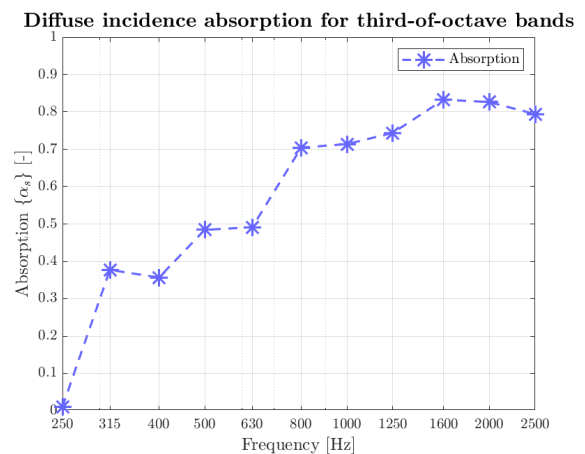


Figure 8: Absorption coefficient per third-of-octave band, calculated from third-of-octave averages of interpolated wave-number spectra.

The graphic displays a rise in absorption following the increase in frequency. This behaviour is coherent to the one expected by the literature [10]. Albeit displaying absorption values seemingly underestimated when compared to techniques such as the reverberation room method, the ensemble average impedance method or the impedance tube method [13], there are a couple of reasons for that. Firstly, Nolan [12] states that, for lower frequencies, the absorption values at large angles of incidence – i.e. increasingly parallel to the absorbing surface or close to the equator – are underestimated because of a “leakage” of incident energy into the region where it is supposed to be only reflected



energy. This phenomenon is related to the limited wave-number resolution utilised. At higher frequencies, there's underestimation of the absorption at shorter angles of incidence – closer to direct incidence – which Nolan [12] refers to being caused by either the spatial resolution of the array or to the angular distribution of the room's normal modes. The combination of these specific underestimations when calculating the average absorption can lead to an underestimation of the coefficient.

5. CONCLUSIONS

The experiment described in this paper was the calculation of the wave-number spectrum and evaluation of the absorption of an interest surface. In this experiment the room had one surface covered with 10.8 m² of glass wool, a porous absorbing material and 150 RTF measurements in random positions were performed. The calculation of the inverse problem's solution and sound field reconstruction were evaluated for a range between the 250 Hz and 2500 Hz third-of-octave bands, with the error limited under 2.5 % between the 400 Hz band and the 1600 Hz band. For lower frequencies the error increases (still under 5 %) due to the spatial resolution of the array. For higher frequencies, the error increases even more (under 10 %) due to the spacing between microphones. The processing was deemed successful.

The plane-wave decomposition allowed the identification of the direct incidence from the sound source over the absorbing sample, the reflection of the surface of interest, and also spurious reflections caused by the other regions of the room and influenced by the position of the sound source. This is important because it is one of the causes of the underestimation of the absorption coefficient values obtained, along with factors influenced by the array characteristics. Nevertheless, the absorption values show an overall behaviour consistent with models present on the literature [10] for that type of material and verified comparing the energy distributions for increasing frequencies.

Overall, the validity of the algorithms for plane wave decomposition was confirmed.

Ideas for future works also stem from the experiments described in this paper, such as the application of plane wave decomposition using other regularization parameters for *in situ* calculation of absorption, implementations of early sound incidence windowing to remove spurious reflections, and the assessment of the decomposition and reconstruction errors of the processing.

6. ACKNOWLEDGMENTS

The authors would like to thank the National Research Council of Brazil (CNPq - *Conselho Nacional de Desenvolvimento Científico e Tecnológico*; *Projeto Universal*: n° 402633/2021-0) and the Coordination for the Improvement of Higher Education Personnel (CAPES - *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior*; CAPES-DS grant: n° 88887.644633/2021-00) for the partial financial support to this research paper.

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